I studied in this thesis the parallel development of the physical concepts and the mathematical formalism used to describe the electromagnetic knowledge in nineteenth and early twentieth century. This study took into account issues such as the ether models used to describe the electromagnetic field, the symmetries associated with the fields, the dimensions of physical quantities. I also studied the choice between quaternions and vectors and the development of the four-dimensional formalism as the best way to describe the electromagnetic phenomena.